

# Cooperative Control of Dual-Arm Concentric Tube Continuum Robots

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# Background & Motivation

# Concentric Tube Continuum Robots (CTCRs)

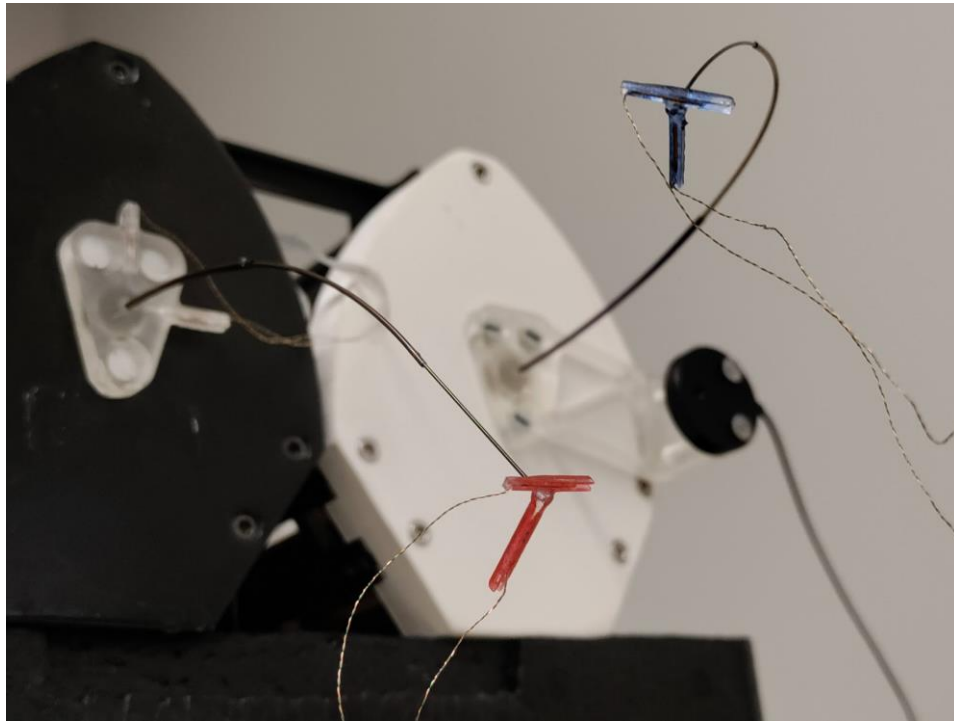


# Dual-Arm Concentric Tube Continuum Robots (DA-CTCRs)



# Overview

## Cooperative control of Dual-Arm Concentric Tube Continuum Robots (DA-CTCRs)



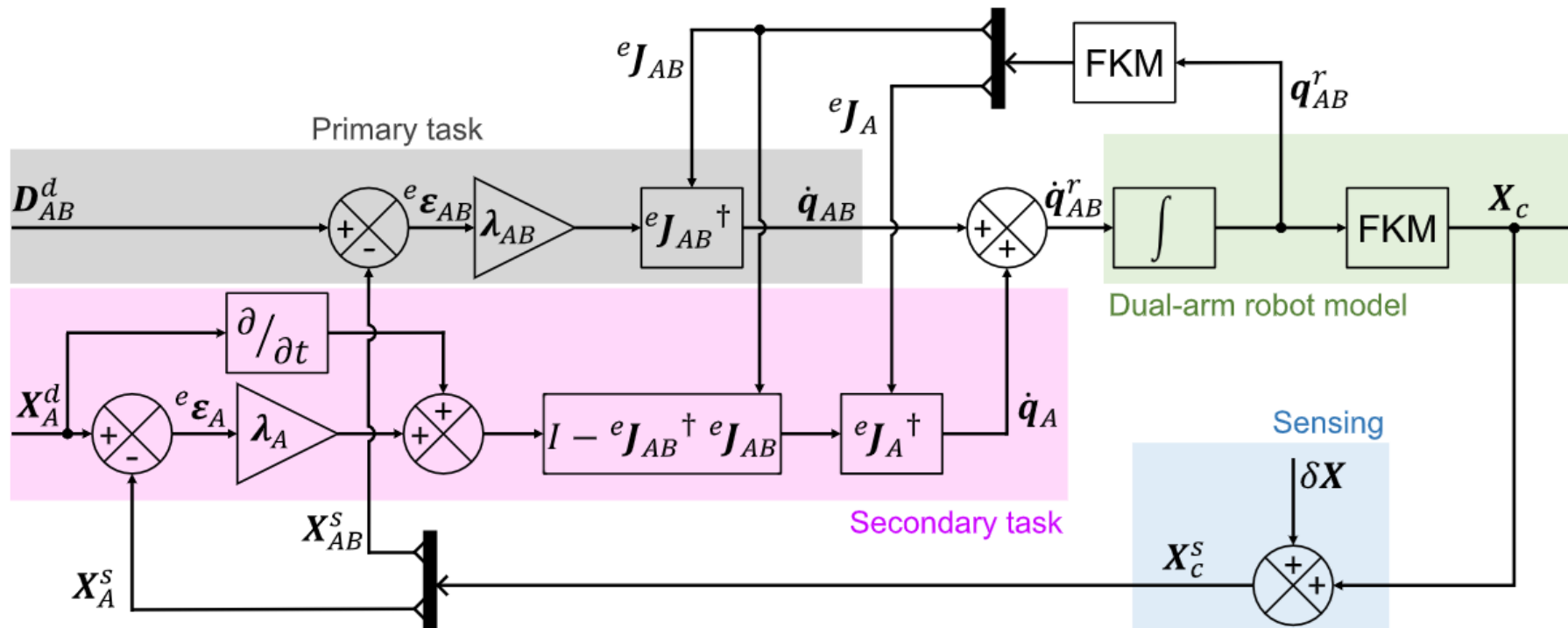
Goal: Provide *automatic assistance* in control of DA-CTCRs.

Contribution: a modular hierarchy-based control framework, with tasks that can be executed based on priority using *redundancy resolution*.

Results: Functionality of *semi-autonomous control* demonstrated in a variety of meaningful scenarios on simulated and real robot models.

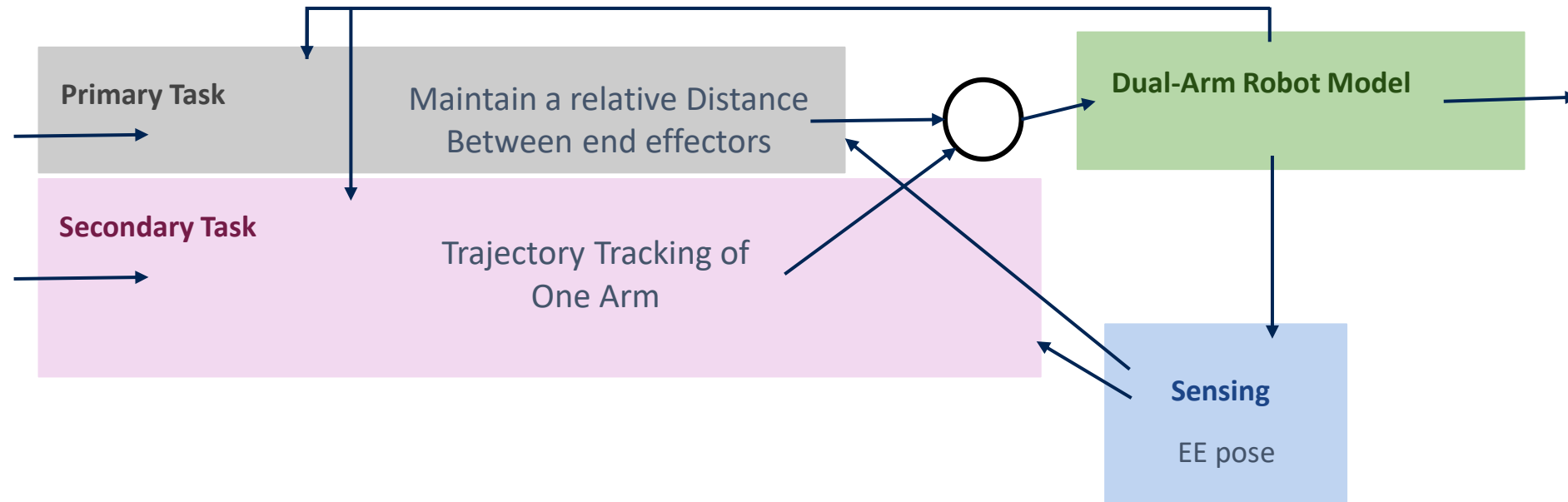
# Control Scheme

# Past Work (Detailed)



[Chikhaoui et al., RAL, 2018, Towards Motion Coordination Control and Design Optimization for Dual-Arm Concentric Tube Continuum Robots]

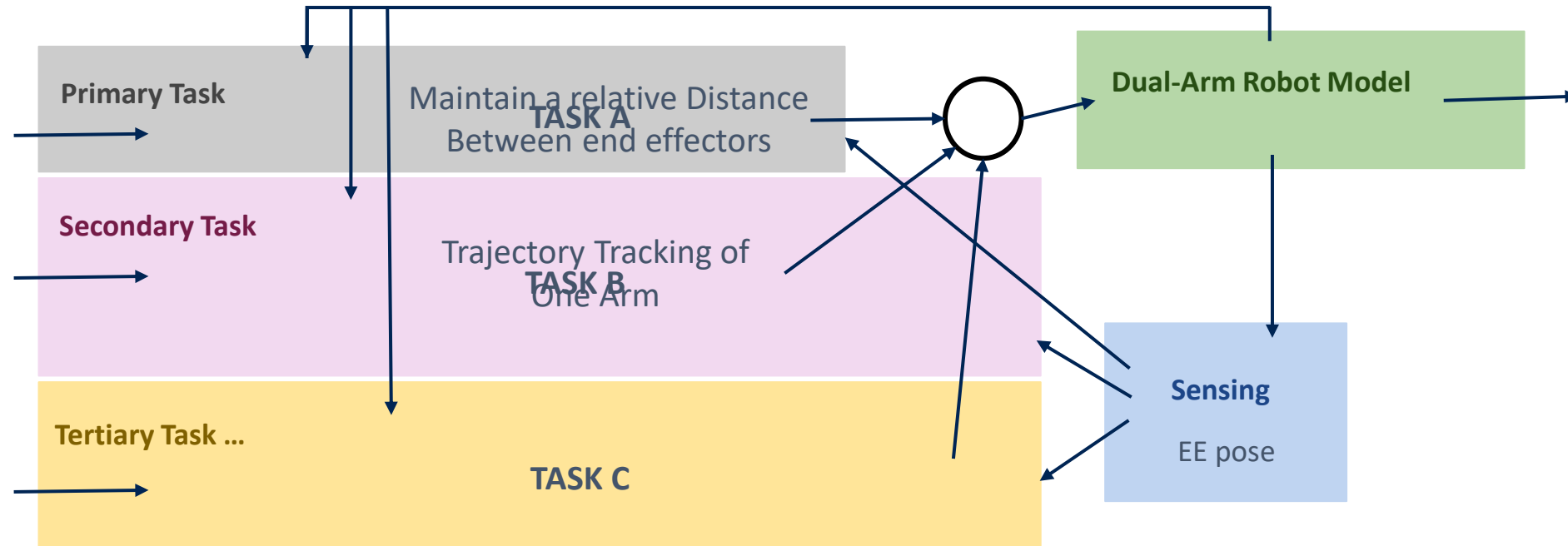
# Past Work (Simplified)



[Chikhaoui et al., RAL, 2018, Towards Motion Coordination Control and Design Optimization for Dual-Arm Concentric Tube Continuum Robots]

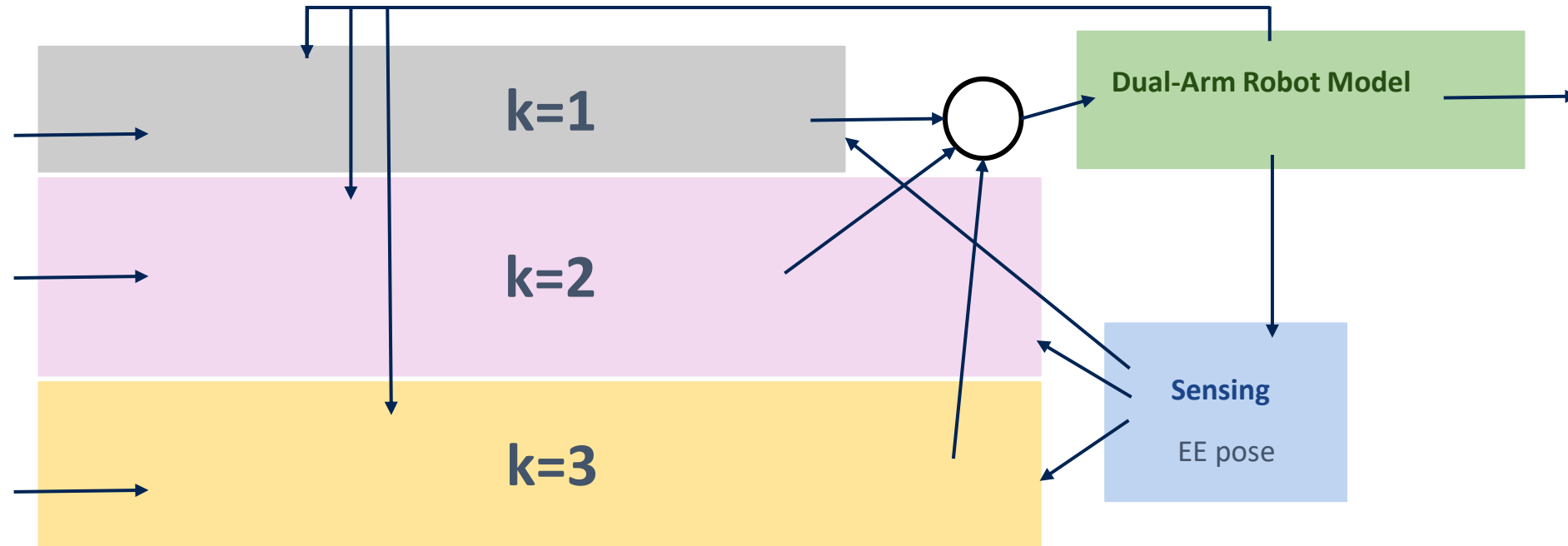


# Proposed Method



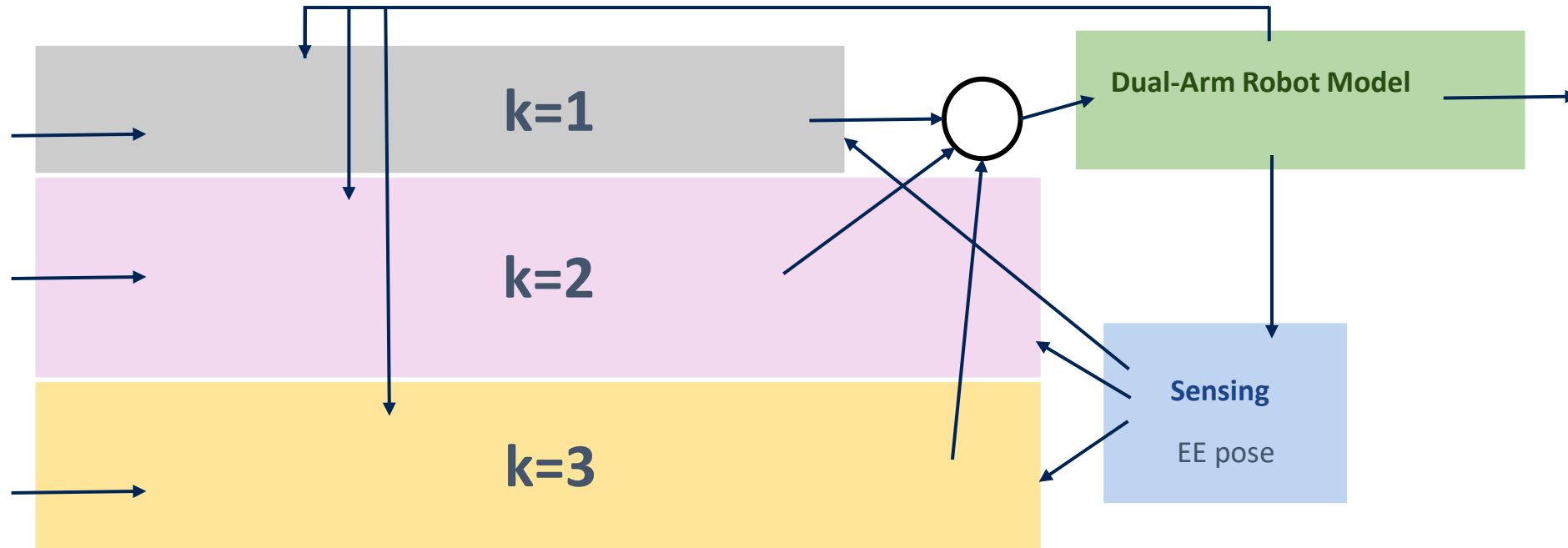
# Task Prioritization

$$\mathbf{P}_k = \mathbf{P}_{k-1} - (\mathbf{J}_k \mathbf{P}_{k-1})^\dagger (\mathbf{J}_k \mathbf{P}_{k-1}) \quad \text{Nullspace Projection}$$



Recursive projection to the nullspace of the prior task Jacobian.

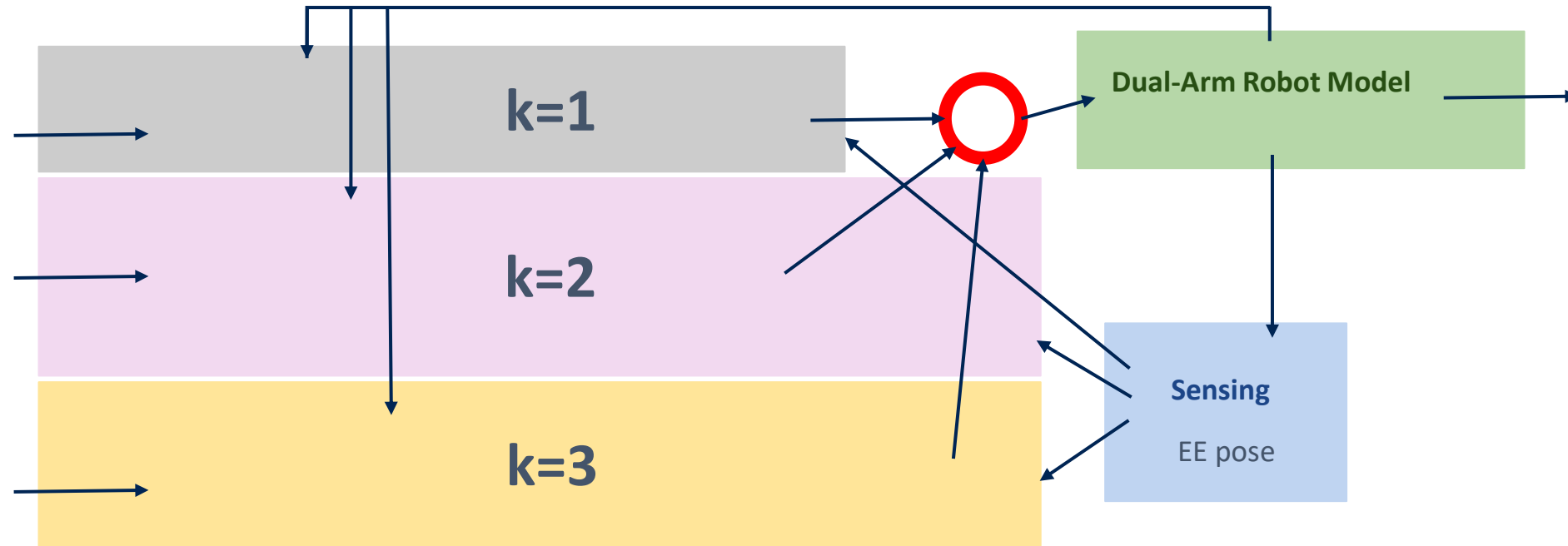
$$\nabla \eta_k(\mathbf{q}) = \begin{cases} \mathbf{J}_k^\dagger \boldsymbol{\epsilon}_k(\mathbf{q}), & \text{for tasks errors} \\ \nabla \mathbf{f}_k(\mathbf{q}), & \text{for task gradients} \end{cases} \quad \text{Contribution of Tasks}$$



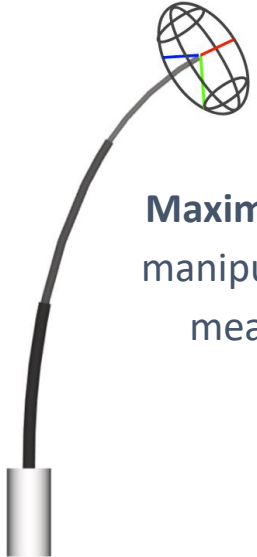
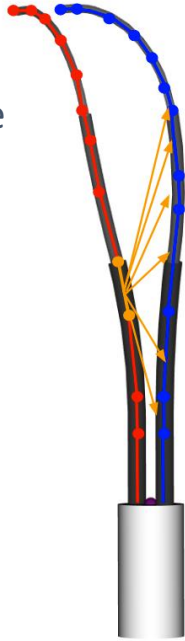
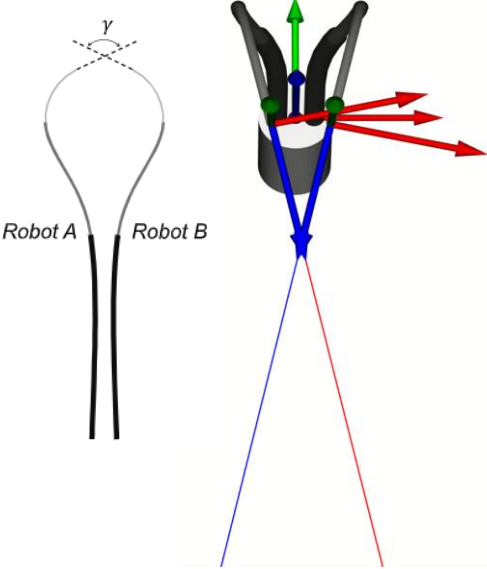
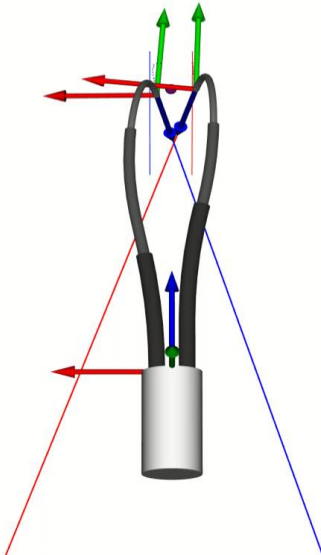
$$\mathbf{J}_k^\dagger = \mathbf{J}_k^T (\mathbf{J}_k \mathbf{J}_k^T)^{-1}$$

$$\dot{\mathbf{q}}_D = \sum_{k=1}^t \lambda_k \mathbf{P}_{k-1} \nabla \eta_k(\mathbf{q})$$

Weighted Sum of  
All Tasks



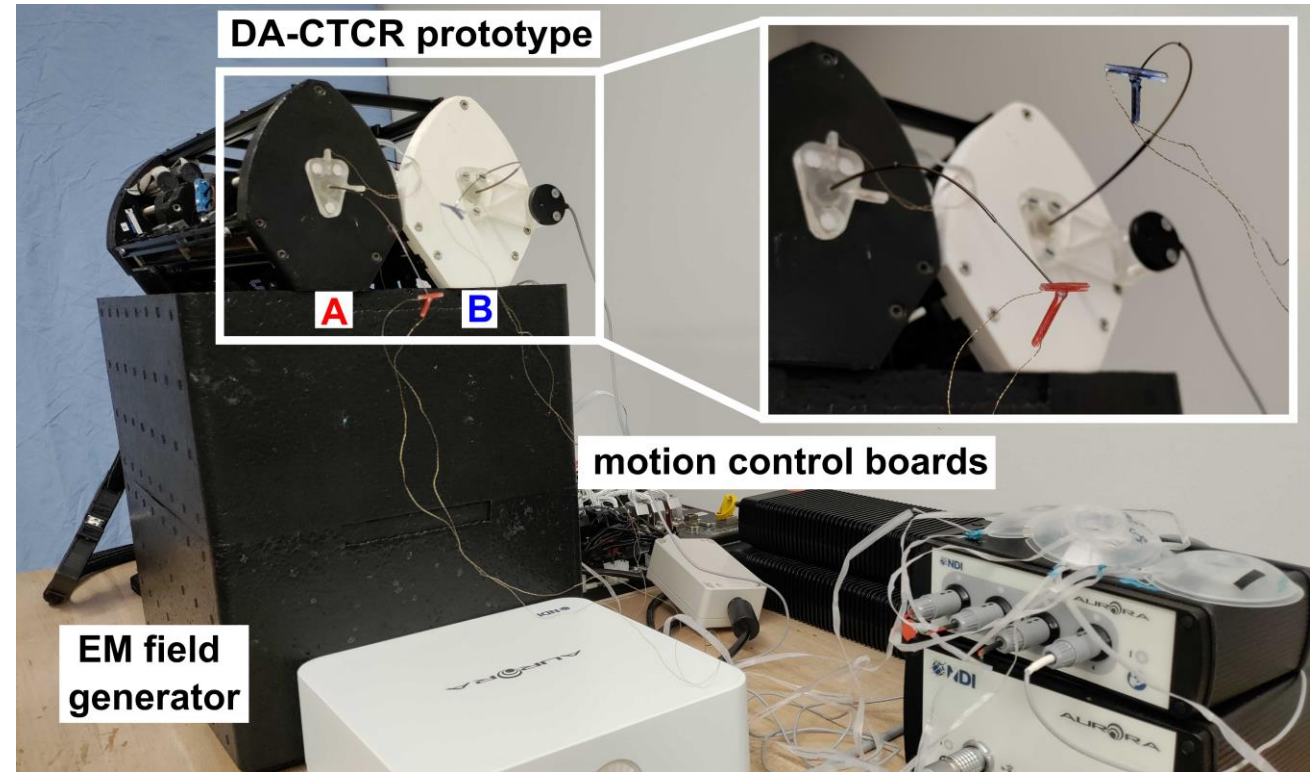
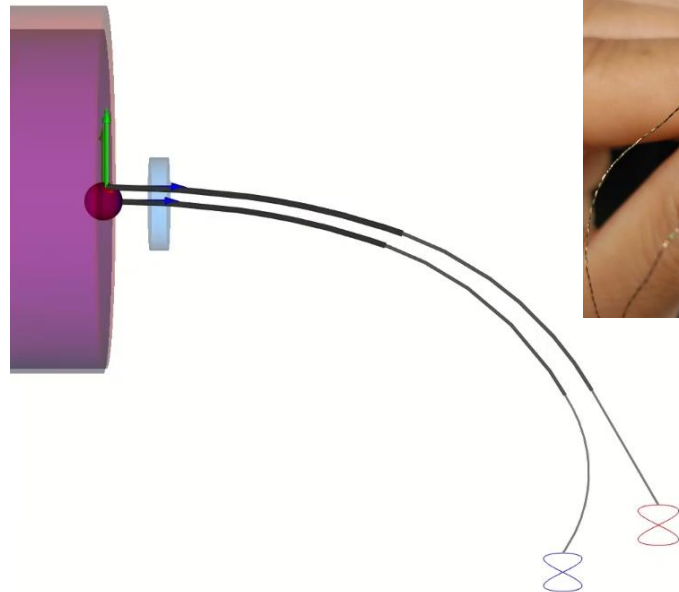
# Task Formulation

Manipulability Maximization	Body Collision Avoidance	Triangulation to a Relative Angle	Triangulation to a Point
 <p>Maximize the manipulability measure</p> $m = \sqrt{\det \left( \mathbf{J}(\mathbf{q}) \mathbf{J}(\mathbf{q})^T \right)}$	<p>Maximize the collision cost  <math>\Rightarrow</math> the distance between the closest 2 backbone segments</p> 	 <p>Minimize the triangulation angle error</p>	<p>Minimize the distance from the line of sight ray to the desired triangulation point</p> 

Control task gradients are computed using finite differences.

# Experimental Evaluation

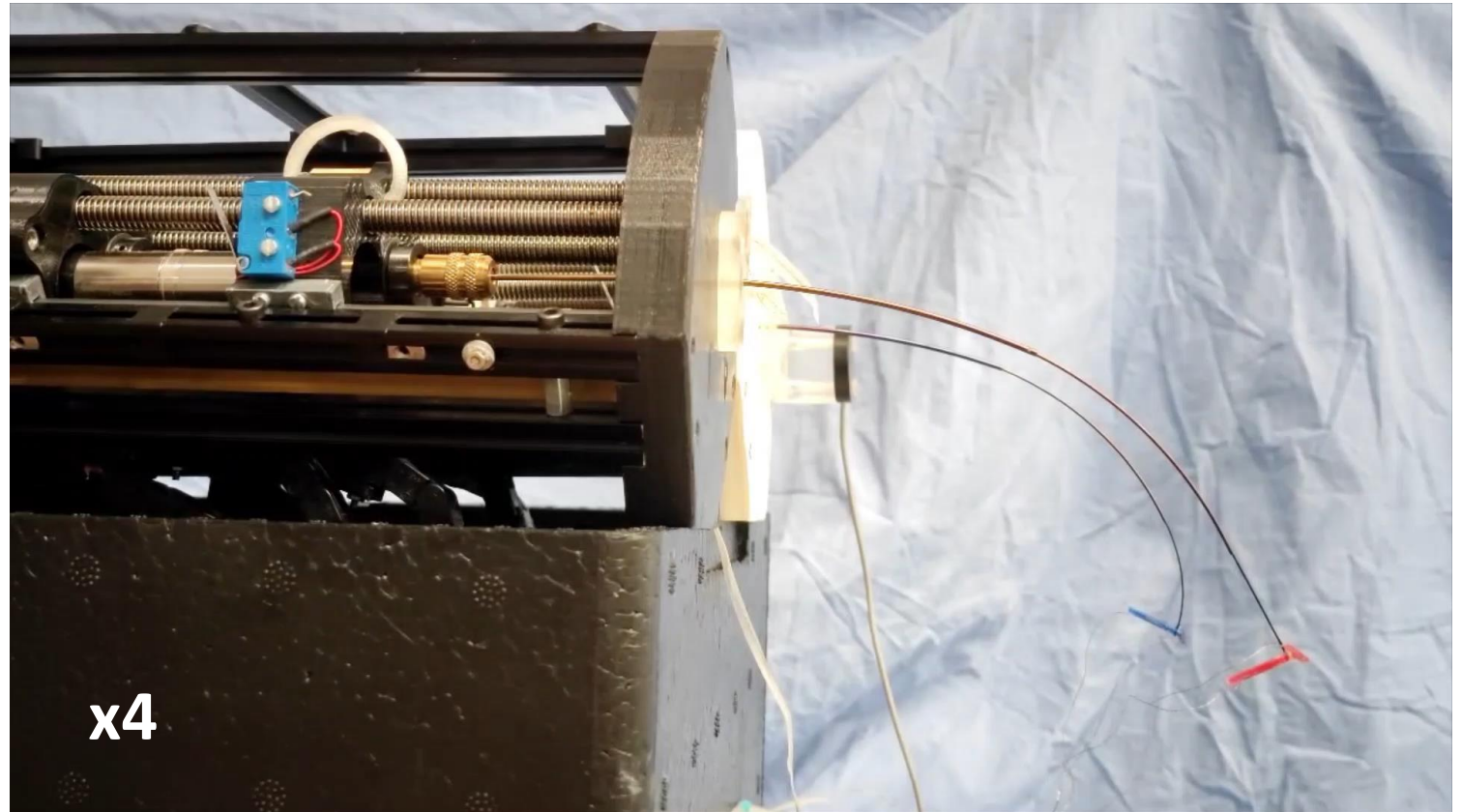
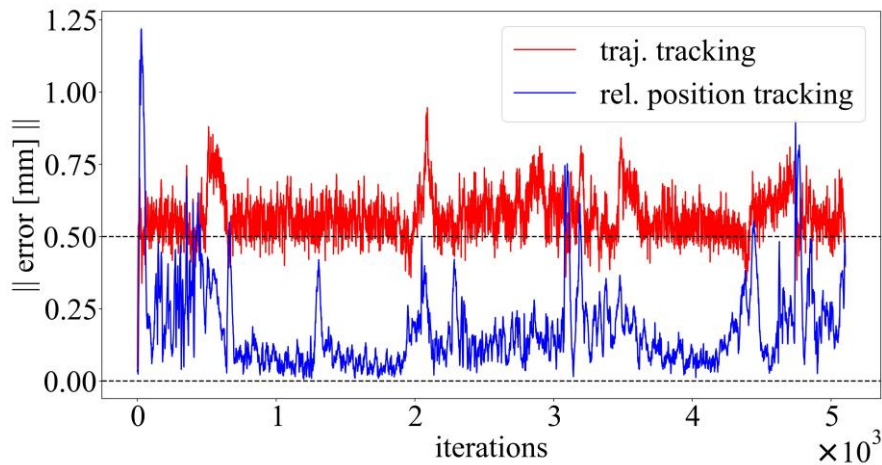
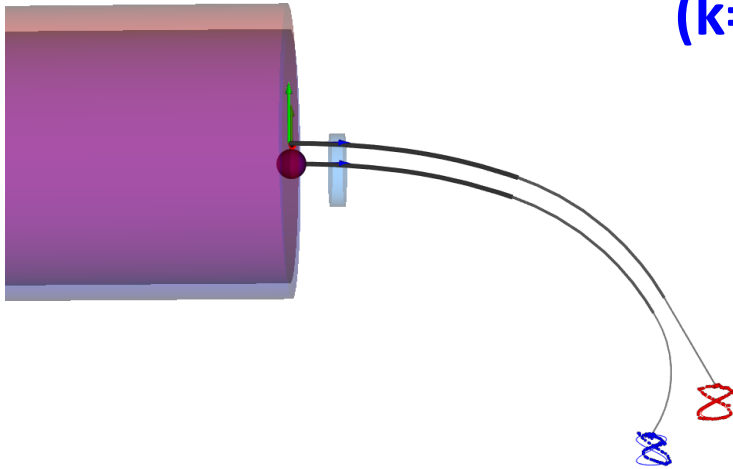
# Physical Setup



# FOLLOWER

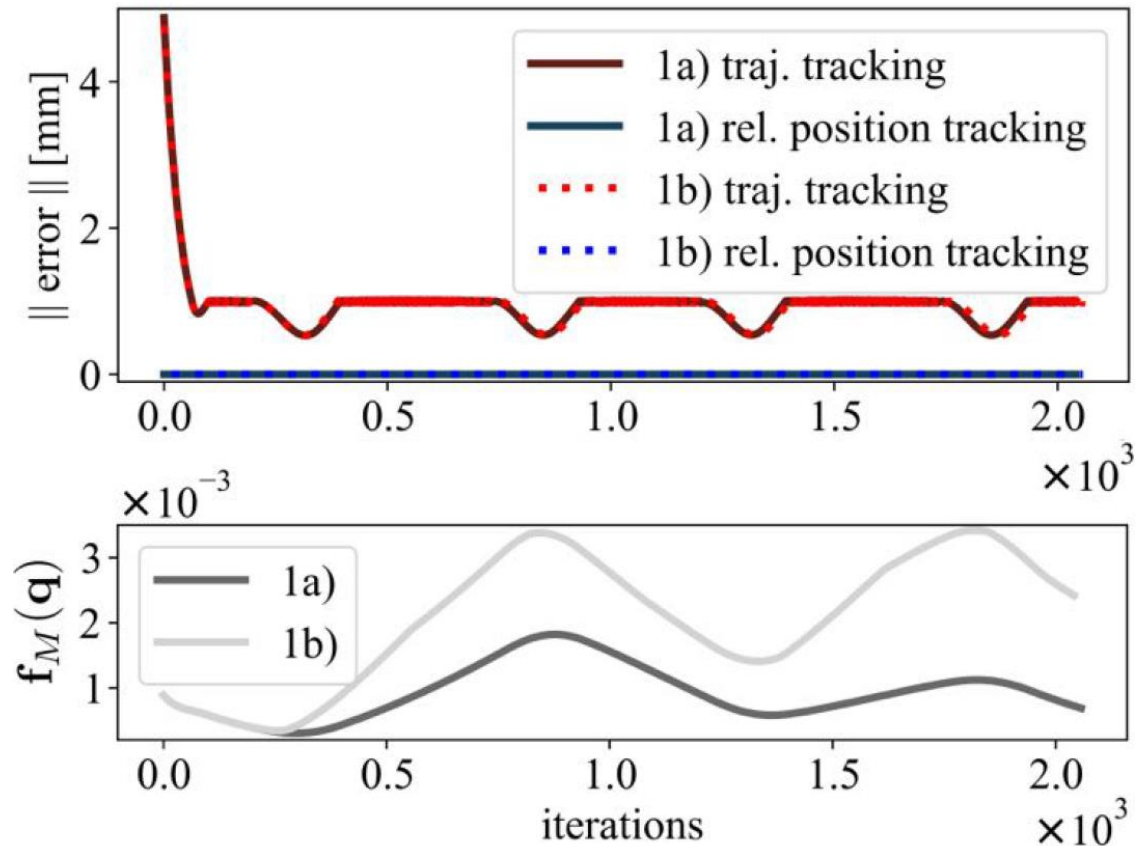
(k=2) Trajectory Tracking of One Arm

(k=1) Maintain a relative Distance Between End Effectors





# Follower + Manipulability Maximization

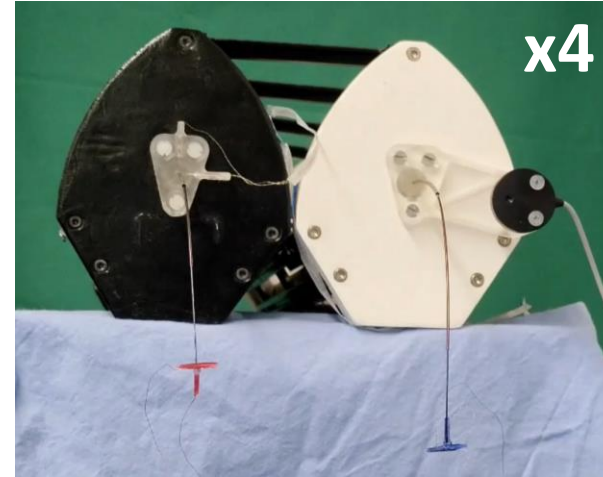
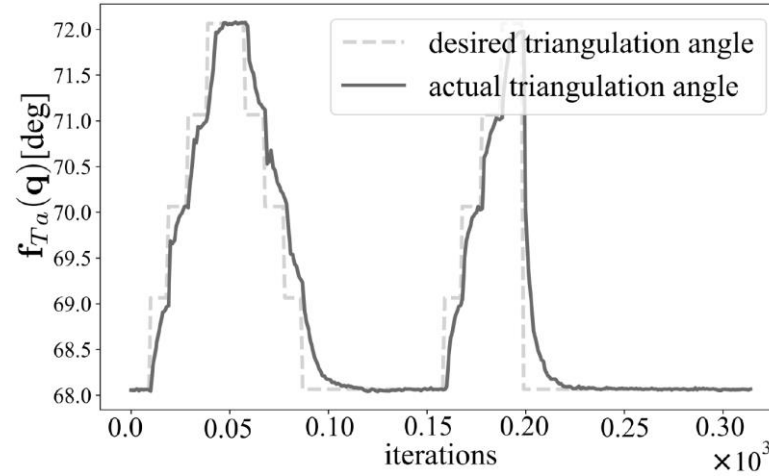


1b)

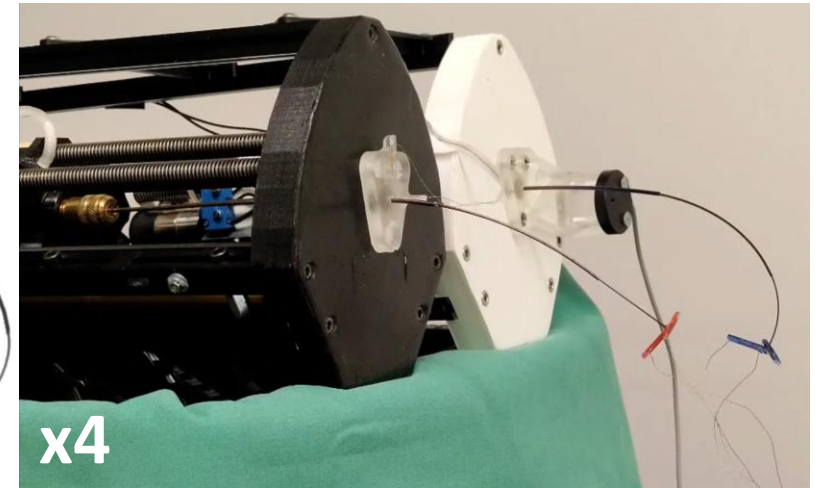
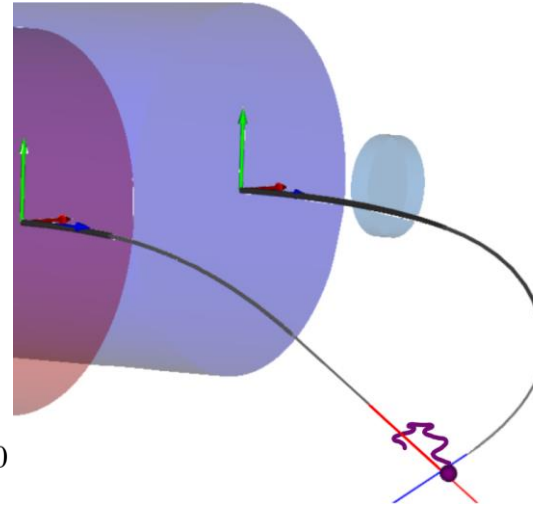
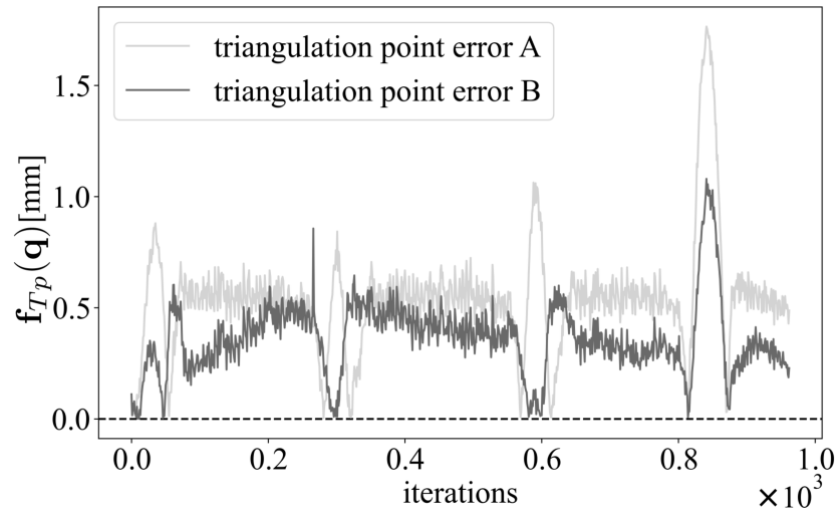
- 1a) (k=3) Manipulability Maximization  
(k=2) Trajectory Tracking of One Arm  
(k=1) Maintain a relative Distance Between End Effectors

# Triangulation to ...

a relative angle



a point



# Conclusion

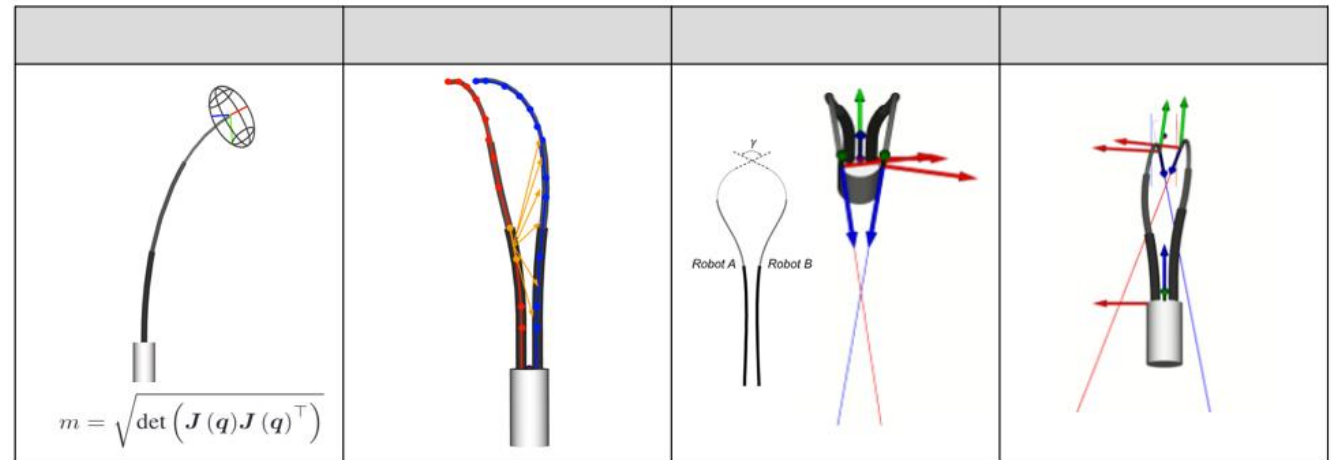
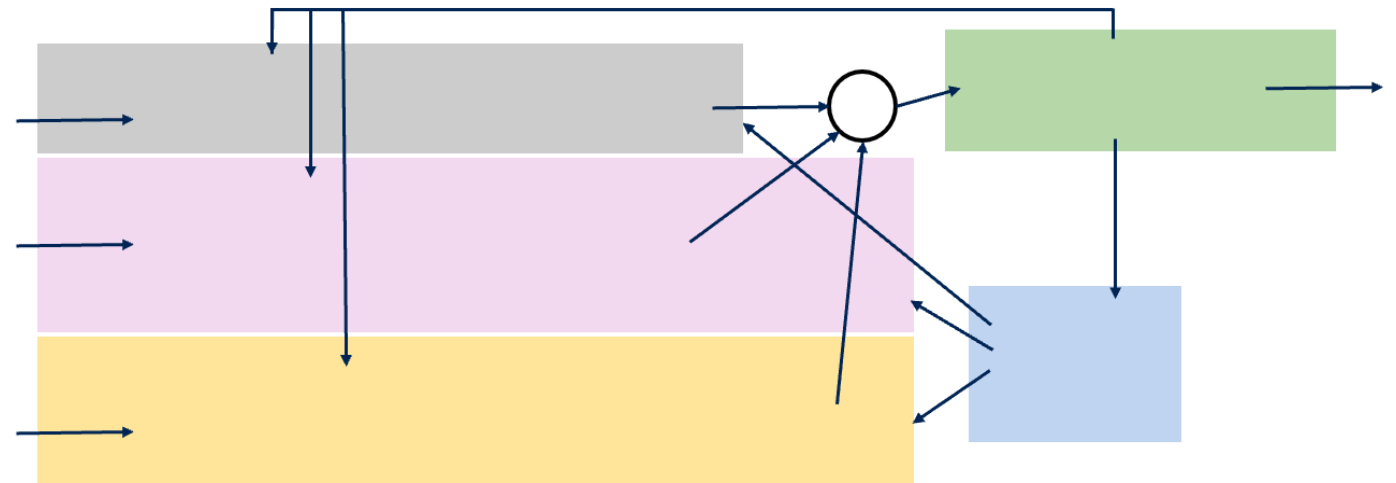
# Summary

Controller is versatile, can prioritize a variety of tasks

→ Can optimize over other quantitative performance indices during control of DA-CTCR

Limitations:

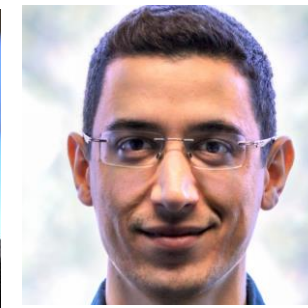
- Finite differences is a local approximation method
- Number of nullspace projections is limited





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